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Extreme Edge-to-vertex Geodesic Graphs

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Abstract

For a connected graph $G = (V, E)$, an edge-to-vertex geodetic basis S in a connected graph G is called an extreme edge-to-vertex geodetic basis if $S \subseteq S_e$, where S_e denotes the set of all extreme edges of G. A graph G is said to be an extreme edge-to-vertex geodesic graph if G contains at least one extreme edge-to-vertex geodetic basis. An edge-to-vertex geodetic basis S in a connected graph G is called a perfect extreme edge-to-vertex geodetic basis if $S = S_e$. A graph G is said to be a perfect extreme edge-to-vertex geodesic graph if G contains a perfect extreme edge-to-vertex geodetic basis, that is, if G has an edge-to-vertex geodetic basis consisting of all the extreme edges of G. Extreme edge-to-vertex geodesic graph G of size q with edge-to-vertex geodetic number q or $q - 1$ or $q - 2$ are characterized. It is shown that for each triple, d, k, q of integers with $2 \leq k \leq q - d + 2$, $d \geq 4$,and $q - d - k + 1 > 0$, there exists a perfect extreme edge-to-vertex geodesic graph G of size q with $diam\ G = d$ and $g_{ev}(G) = k$.

Keywords: distance, geodesic, edge-to-vertex geodetic basis, edge-to-vertex geodetic number.

AMS Subject Classification: 05C12.

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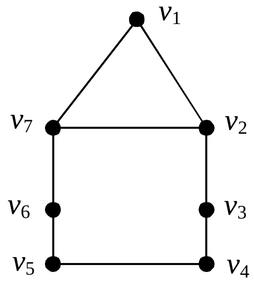
1. Introduction

By a graph $G = (V, E)$, we mean a finite undirected connected graph without loops or multiple edges. The *order* and *size* of G are denoted by p and q respectively. We consider connected graphs with at least three vertices. For basic definitions and terminologies we refer to [1, 4]. A subset $M \subseteq E(G)$ is called a matching of G if no pair of edges in M are incident. The maximum size of such M is called the matching number of G and is denoted by $\propto'(G)$. An edge covering of G is a subset $K \subseteq E(G)$ such that each vertex of G is end of some edge in K. The number of edges in a minimum edge covering of G, denoted by $ß'(G)$ is the edge covering number of G. For vertices u and v in a connected graph G, the distance $d(u, v)$ is the length of a shortest $u - v$ path in G. An $u - v$ path of length $d(u, v)$ is called an $u - v$ *geodesic*. For a vertex v of G, the eccentricity $e(v)$ is the distance between v and a vertex farthest from v. The minimum eccentricity among the vertices is the radius, *rad G* and the maximum eccentricity is the diameter, *diam G* of G. A *geodetic set* of G is a set S of vertices such that every vertex of G is contained in a geodesic joining some pair of vertices of S. The *geodetic number* $g(G)$ of G is the minimum cardinality of its geodetic sets and any geodetic set of cardinality $g(G)$ is a minimum *geodetic set* or simply a g-set of G. The geodetic number of a graph was introduced in [1] and further studied in [2,5]. It was shown in [5] that determining the geodetic number of a graph is an NP-hard problem. $N(v) = \{ u \in V(G) : uv \in E(G)\}$ is called the neighborhood of the vertex v in G. A vertex v is an *extreme vertex* of a graph G if the subgraph induced by its neighbors is complete. The number of extreme vertices in G is its *extreme order* ex(G). A graph G is said to be an *extreme geodesic graph* if $g(G) = \text{ex}(G)$, that is if G has a unique minimum geodetic set consisting of the extreme vertices of G. The concept of extreme geodesic graphs is introduced in [3]. For subsets A and B of $V(G)$, the *distance* $d(A, B)$ is defined as $d(A, B) = \min\{d(x, y) : x \in A, y \in B\}$. An $u - v$ path of length $d(A, B)$ is called an $A - B$ *geodesic* joining the sets A, B, where $u \in A$ and $v \in B$. A vertex x is said to lie on an $A - B$ geodesic if x is a vertex of an $A - B$ geodesic. For $A = \{u, v\}$ and $B = \{z, w\}$ with uv and zw edges, we write an $A - B$ geodesic as $uv - zw$ geodesic and $d(A, B)$ as $d(uv, zw)$. A set $S \subseteq E(G)$ is called an *edge-to-vertex geodetic set* if every vertex of G is either incident with an edge of S or lies on a geodesic joining a pair of edges of S. The *edge-to-vertex geodetic number* $g_{ev}(G)$ of G is the minimum cardinality of its edge-to-vertex geodetic sets and any edge-to-vertex geodetic set of cardinality $g_{ev}(G)$ is an *edge-to-vertex geodetic basis* of G. The edge-to-vertex geodetic number of a graph was introduced in [9] and further studied in [6,8]. Since every edge covering of G is an edge-to-vertex geodetic set of G, we have $g_{ev}(G) \leq ß'(G)$. For an edge $e = uv \in E(G)$, $N(e) = N(u) \cup N(v)$. For a set $S \subseteq E(G)$, $N(S) = \{ N(e): e \in S\}$. An edge e of a graph G is called an *extreme edge* of G if one of its ends is an extreme vertex of G. Let S_e denotes the set of all extreme edges of G, $E(e)$ denotes the number of extreme edges of G, and $c(G)$ denotes the length of the longest cycle in G. A double star is a tree with diameter three. A *caterpillar* is a tree or more, for which the removal of all end-vertices leaves a path.

Example 1.1. For the graph G given in Figure 1.1 with $A = \{v_4, v_5\}$ and $B = \{v_1, v_2, v_7\}$, the paths $P : v_5, v_6, v_7$ and $Q : v_4, v_3, v_2$ are the only two $A - B$

geodesics so that $d(A, B) = 2$.

Example 1.2. For the graph G given in Figure 1.2, the three $v_1v_6 - v_3v_4$ geodesics are $P : v_1, v_2, v_3$; $Q : v_1, v_2, v_4$; and $R : v_6, v_5, v_4$ with each of length 2 so that $d(v_1v_6, v_3v_4) = 2$. Since the vertices v_2 and v_5 lie on the $v_1v_6 - v_3v_4$ geodesics P and R respectively, $S = \{v_1v_6, v_3v_4\}$ is an edge-to-vertex geodetic basis of G so that $g_{ev}(G) = 2$.



G

Figure 1.1

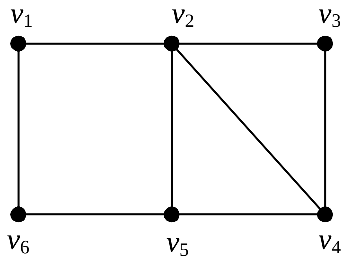


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Figure 1.2

The following theorems are used in sequel.

Theorem 1.1.[9] If v is an extreme vertex of a connected graph G, then every edge-to-vertex geodetic set contains at least one extreme edge is incident with v.

Theorem 1.2.[9] For any connected graph G, $g_{ev}(G) = q$ if and only if G is a star.

Theorem 1.3. [9] For any connected graph G with size $q \geq 3$, $g_{ev}(G) = q - 1$ if and only if G is either a double star or C_3.

Theorem 1.4.[9] For a non-trivial tree T with k end-vertices, $g_{ev}(T) = k$.

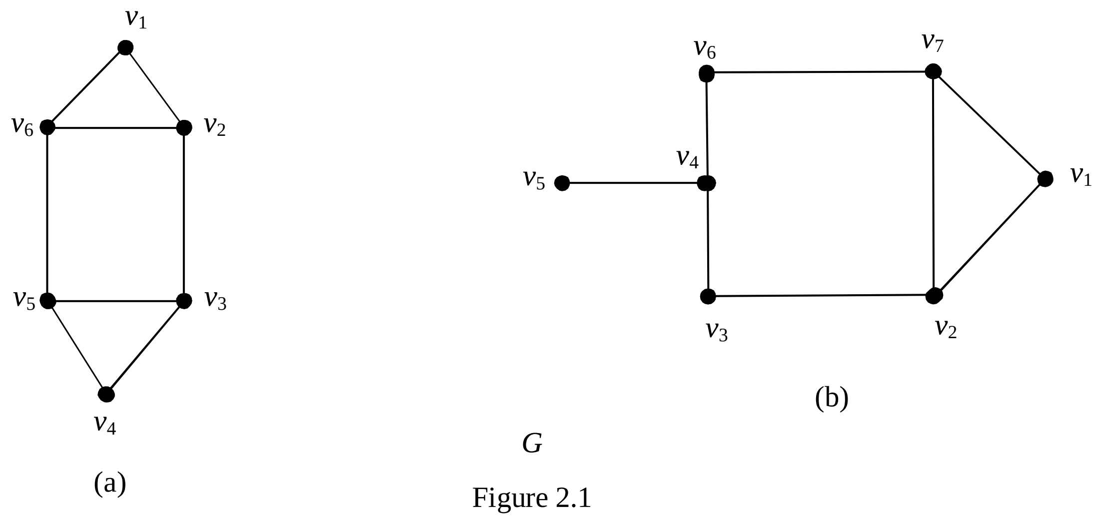
Theorem 1.5. [9] For any graph G of order p, $g_{ev}(G) \leq p - \alpha'(G)$.

2. Extreme Edge-to- Vertex Geodesic Graphs

Definition 2.1. An edge-to-vertex geodetic basis S in a connected graph G is called an *extreme edge-to-vertex geodetic basis* if $S \subseteq S_e$. A graph G is said to be an *extreme edge-to-vertex geodesic graph* if G contains at least one extreme edge-to-vertex geodetic basis. An edge-to-vertex geodetic basis S in a connected graph G is called a *perfect extreme edge-to-vertex geodetic basis* if $S = S_e$. A graph G is said to be a *perfect extreme edge-to-vertex geodesic graph* if G contains a perfect extreme edge-to-vertex geodetic basis, that is, if G has an edge-to-vertex geodetic basis consisting of all the extreme edges of G.

Example 2.2. For the graph G given in Figure 2.1(a), $S_e = \{v_1v_2, v_1v_6, v_3v_4, v_4v_5\}$. The set $S_1 = \{v_1v_2, v_4v_5\}$ is an edge-to-vertex geodetic basis of G. Since $S_1 \subseteq S_e$, S_1 is an extreme edge-to-vertex geodetic basis of G. Therefore, G is an extreme edge-to-vertex geodesic graph. For the graph G given in Figure 2.1(b), $S_e = \{v_1v_2, v_1v_7, v_4v_5\}$ is the unique extreme edge-to-vertex geodetic basis of G so that $g_{ev}(G) = 3 = E(e)$. Therefore G is a perfect extreme edge-to-vertex geodesic graph.

Remark 2.3. For an extreme edge-to-vertex geodesic graph G, there can be more than one extreme edge-to-vertex geodetic basis. For the graph G given in Figure 2.1(a), $S_2 = \{v_1v_6, v_3v_4\}$ is an extreme edge-to-vertex geodetic basis.



G

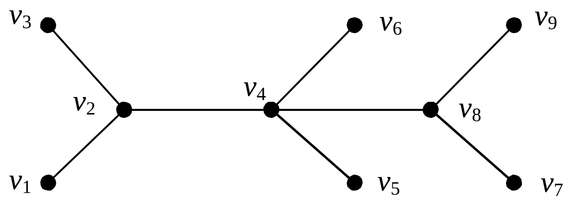
Figure 2.1

For the complete graph $G = K_p(p \geq 3)$, every edge is an extreme edge. In [9], it is proved that, $g_{ev}(K_p)$ is either $p/2$ or $(p+1)/2$. So K_p is an extreme edge-to-vertex geodesic graph. Since $g_{ev}(K_p) \neq E(e)$, K_p is not a perfect extreme edge-to-vertex geodesic graph. A nontrivial tree T has k extreme edges , namely its end edges and so $E(e) = k$. Since $g_{ev}(G) = k$, it follows that T is a perfect extreme edge-to-vertex geodesic graph. Obviously, a cycle $C_p(p \geq 4)$ has no extreme edges, a cycle is not an extreme edge-to-vertex geodesic graph. For any complete bipartite graph $G = K_{m,n}(2 \leq m \leq n)$, it is easily to see that no edge is an extreme edge and so G is not an extreme edge-to-vertex geodesic graph.

Theorem 2.4. Let G be an extreme edge-to-vertex geodesic graph of size $q \geq 2$ such that $d(e, f) = 0$ or 1 for every $e, f \in E(G)$. Then $g_{ev}(G) = \beta'(G)$.

Proof. Let S be an edge-to-vertex geodetic basis of G and $v \in V(G)$. We claim that v is incident with an edge of S. If not, then by Theorem 1.1, v is not an extreme vertex of G. If $v \notin N(S)$, then v lies on a xu- yw geodesic, where $xu, yw \in S$. Then it follows that $d(xu, yw) \geq 2$, which is a contradiction. Therefore $v \in N(S)$. Since S is an edge-to-vertex geodetic basis of G and since $d(e, f) = 0$ or 1 for every $e, f \in E(G)$, the only geodesics containing v are xvy and $xyvw$, where $xv, vy, xy, vw \in S$. This contradicts the fact that v is not incident with an edge of S. Therefore v is incident with an edge of S. Which implies that S is an edge covering of G and so $ß'(G) \leq g_{ev}(G)$. Hence $g_{ev}(G) = ß'(G)$. ■

Remark 2.5. The converse of the Theorem 2.4 is not true. For the extreme edge-to-vertex geodesic graph G given in Figure 2.2, $g_{ev}(G) = ß'(G) = 6$ and $d(v_1v_2, v_8v_9) \geq 2$.



G

Figure 2.2

Theorem 2.6. Let G be a connected graph of size $q \geq 2$. Then G is a perfect extreme edge-to-vertex geodesic graph with edge-to-vertex geodetic number q if and only if $G = K_{1,}q$.

Proof. This follows from Theorem 1.2.

Theorem 2.7. Let G be a connected graph of size $q \geq 3$. Then G is an extreme edge-to -vertex geodesic graph with edge-to-vertex geodetic number $q - 1$ if and only if G is either C_3 or a double star.

Proof. This follows from Theorem 1.3 ■

Theorem 2.8. If G is an extreme edge-to-vertex geodesic graph of size $q \geq 4$ and not a tree such that $g_{ev}(G) = q - 2$, then G is unicyclic and $c(G) = 3$.

Proof. Let G have more than one cycle. Then $q \geq p + 1$ and so $p - 1 \leq q - 2 = g_{ev}(G) \leq p - \propto'(G)$, by Theorem 1.5. Hence $\propto'(G) = 1$ and so G must be either a star or the cycle C_3, a contradiction. Therefore G is unicyclic. Then it follows from Theorem 1.5, $\propto'(G) \leq 2$. Let C_k be the unique cycle of G. We have $k \leq 5$ since otherwise $\propto'(G) \geq \propto'(C_k) \geq 3$. Therefore we have the following three cases:

Case 1. $k = 5$. Then G cannot have any other vertices since otherwise $\propto'(G) \geq 3$. Therefore $G = C_5$ which is not an extreme edge-to-vertex geodesic graph, which is a contradiction.

Case 2. $k = 4$. If $G = C_4$, then G is not an extreme edge-to-vertex geodesic graph. So let $G \neq C_4$. Because $\propto'(G) \leq 2$, only one of the vertices of C_4 has degree more than 2. Therefore G is not an extreme edge-to-vertex geodesic graph, which is a contradiction. Therefore $c(G) = 3$ ■

Theorem 2.9. Let G be a connected graph of size $q \geq 4$. Then G is an extreme edge-to-vertex geodesic graph with edge-to-vertex geodetic number q-2 if and only if $G = K_{1,}\, q - 1 + e$ or caterpillar with diameter 4 or the graph G given in Figure 2.3.

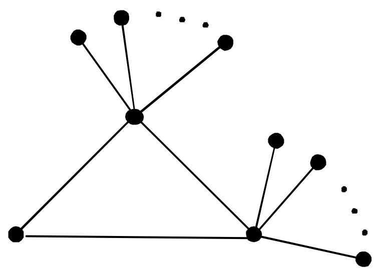


Figure 2.3

Proof. For a caterpillar of diameter 4, the result follows from Theorem 1.4. For $G = K_{1,}q - 1 + e$, it follows from Theorem 1.1, that the set of all end edges of G together with e forms an edge -to-vertex geodetic basis so that $g_{ev}(G) = q - 2$. Further it is easily verified that $g_{ev}(G) = q - 2$ for the graph given in Figure 2.3.

Conversely let G be an extreme edge-to-vertex geodesic graph such that $g_{ev}(G) = q - 2$. Then by Theorem 2.8, G is either a tree or unicyclic. Let G be a tree. Then it follows from Theorem 1.4 that G has just two internal edges and hence G is a caterpillar. Thus in this case the graph reduces to a caterpillar of diameter 4. Now, let G be an unicyclic. By Theorem 2.8, $c(G) = 3$. Since $g_{ev}(C_3) = 2 = q - 1$, we have $G \neq C_3$. Let $V(C_3) = \{v_1, v_2, v_3\}$. We note that if $u \in V(G) - V(C_3)$, then deg $u = 1$. Otherwise, there are $u_1, u_2 \in V(G) - V(C_3)$ such that u_1 is adjacent to both u_2 and v_1, say. Then it is easily seen that $E(G) - \{u_1v_1, v_1v_2, v_1v_3\}$ is an edge-to-vertex geodetic set, which implies that $g_{ev}(G) \leq q - 3$. Further at least one of v_i`s should be of degree 2. Otherwise $E(G) - E(C_3)$ is an edge-to-vertex geodetic set, which is impossible. Thus G should be either $K_{1,}q - 1 + e$ or a graph like Figure 2.3. ■

The following theorem is proved in [9].

Theorem A. Let G be a connected graph of size q and diameter d, then $g_{ev}(G) \leq q - d + 2$.

If G is a perfect extreme edge-to-vertex geodesic graph, then we have the following result.

Theorem 2.10. If G is a perfect extreme edge-to-vertex geodesic graph of size q and diameter d, then $E(e) \leq q\text{-}d\text{+}2$.

Proof. Since G is a perfect extreme edge-to-vertex geodesic graph, we have $g_{ev}(G) = E(e)$, now the result follows from Theorem *A*. ■

The following theorem characterize for trees.

Theorem 2.11. For any tree T, $g_{ev}(T) = q - d + 2 = E(e)$ if and only if T is a caterpillar.

Proof. Let $P : v_0, v_1, ..., v_{d-1}, v_d = v$ be a diametral path of length d . Let $e_i = v_{i-1}v_i$ $(1 \leq i \leq d)$ be the edges of the diametral path P. Let k be the number of end edges of T and l be the number of internal edges of T other than e_i $(2 \leq i \leq d - 1)$. Then $d - 2 + l + k = q$. By Theorem 1.4, $g_{ev}(T) = k = E(e)$ and so $g_{ev}(T) = q - d + 2 - l$. Hence $g_{ev}(T) = q - d + 2 = E(e)$ if and only if $l = 0$, if and only if all internal vertices of T lie on the diametral path P, if and only if T is a caterpillar. ■

In the following we give some realization results on perfect extreme edge-to-vertex geodesic graphs.

Theorem 2.12. For every pair k, q of integers with $2 \leq k \leq q$, there exists a perfect extreme edge-to-vertex geodesic graph of size q with edge-to-vertex geodetic number q.

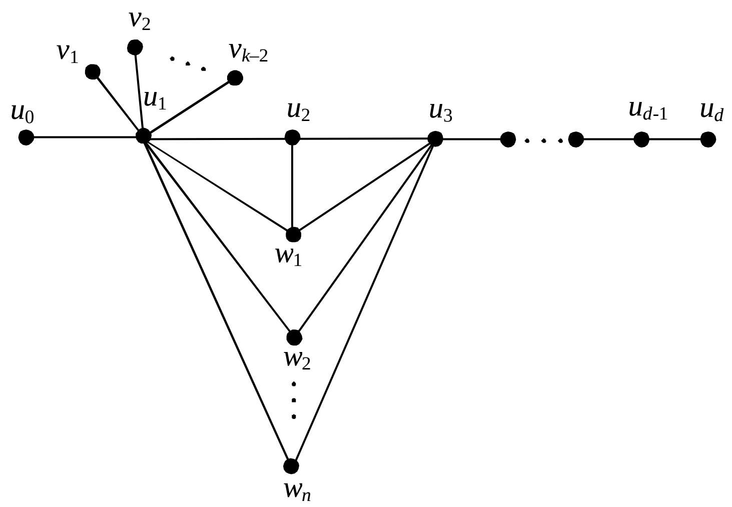
Proof. For $k = q$, the result follows from Theorem 2.6. Also, for each pair of integers with $2 \leq k < q$, there exists a tree of size q with k end edges. Hence the result follows from Theorem 1.4.

Theorem 2.13. For each triple, d, k, q of integers with $2 \leq k \leq q - d + 2$,$d \geq 4$, and $q - d - k + 1 > 0$, there exists a perfect extreme edge-to-vertex geodesic graph G of size q with *diam* $G = d$ and $g_{ev}(G) = k$.

Proof. Let $2 \leq k = q - d + 2$. Let G be the graph obtained from the path P of length d by adding $q - d$ new vertices to P and joining them to any cut-vertex of P. Then G is a tree of size q and *diam* $G = d$. By Theorem 1.4, $g_{ev}(G) = q - d + 2 = k$. Now, let $2 \leq k < q - d + 2$.

Case 1. $q - d - k + 1$ is even. Let $(q - d - k + 1) \geq 2$. Let $n = \frac{(q-d-k+1)}{2}$. Then $n \geq 1$. Let $P_d : u_0, u_1, \ldots, u_d$ be a path of length d. Add new vertices $v_1, v_2, \ldots, v_{k-2}$ and $w_1, w_2, \ldots, w_n$ and join each v_i $(1 \leq i \leq k - 2)$ with u_1 and also join each w_i $(1 \leq i \leq n)$ with u_1 and u_3 in P_d. Now, join w_1 with u_2 and we obtain the graph G in Figure 2.4(a). Then G has size q and diameter d. By Theorem 1.1**,** all the end-edges u_1v_i $(1 \leq i \leq k - 2)$, u_0u_1 and u_{d-1} u_d lie in every edge-to-vertex geodetic set of G. Let $S = \{u_1v_1, u_1v_2, \ldots, u_1v_{k-2}, u_1u_0, u_{d-1}u_d\}$ be the set of all end-edges of G. Then it is clear that S is an extreme edge-to-vertex geodetic set of G and so $g_{ev}(G) = k$. Therefore G is a perfect extreme edge-to-vertex geodesic graph.

Case 2. $q - d - k + 1$ is odd. Let $q - d - k + 1 \geq 5$. Let $m = (q - d - k) / 2$. Then $m \geq 2$. Let $P_d : u_0, u_1, \ldots, u_d$ be a path of length d. Add new vertices $v_1, v_2, \ldots, v_{k-2}$ and $w_1, w_2, \ldots, w_m$ and join each v_i $(1 \leq i \leq k - 2)$ with u_1 and also join each w_i $(1 \leq i \leq m)$ with u_1 and u_3 in P_d. Now join w_1 and w_2 with u_2 and we obtain the graph G in Figure 2.4(b). Then G has size q and diameter d. Now, as in Case 1, $S = \{u_1v_1, u_1v_2, \ldots, u_1v_{k-2}, u_0u_1, u_{d-1}u_d\}$ is an extreme edge-to-vertex geodetic set of G so that $g_{ev}(G) = k$. Therefore G is a perfect extreme edge-to-vertex geodesic graph.



G

Figure 2.4(a)

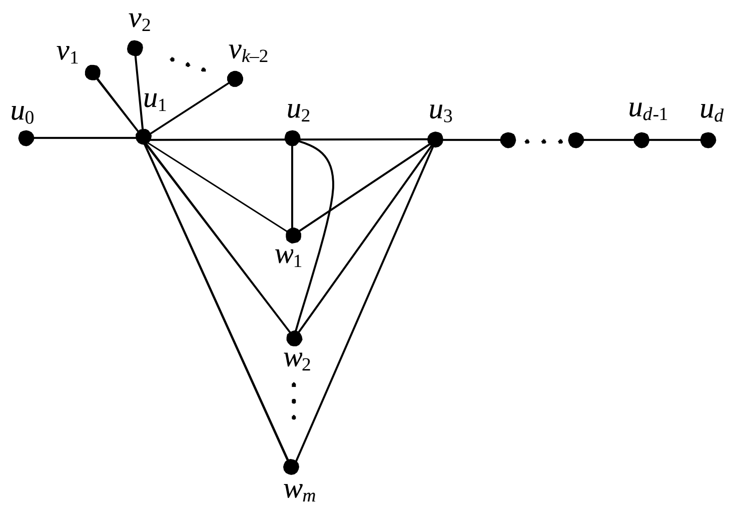


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Figure 2.4(b)

Let $q - d - k + 1 = 1$. Let $P_d : u_0, u_1, \ldots, u_d$ be a path of length d. Add new vertices $v_1, v_2, \ldots, v_{k-2}$ and w_1 and join each v_i $(1 \le i \le k - 2)$ with u_1 and also join w_1 with u_1 and u_3 in P_d, there by obtaining the graph G in Figure 2.4(c). Then the graph is of size q and diameter d. Now, as in Case 1, $S = \{u_1v_1, u_1v_2, \ldots, u_1v_{k-2}, u_0u_1, u_{d-1}u_d\}$ is an extreme edge-to-vertex geodetic set of G so that $g_{ev}(G) = k$. Therefore G is a perfect extreme edge-to-vertex geodesic graph.

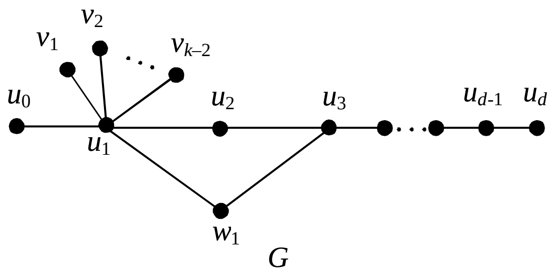

Figure 2.4(c)

Now, let $q - d - k + 1 = 3$. Let P_d : $u_0, u_1, \ldots, u_d$ be a path of length d. Add new vertices $v_1, v_2, v_3, \ldots, v_{k-2}, w_1$ and w_2 and join each v_i $(1 \leq i \leq k - 2)$ with u_1 and also join w_1 and w_2 with u_1 and u_3 and obtain the graph G in Figure 2.4(d). Then G has size q and diameter d. Now, as in Case 1, $S = \{u_1v_1, u_1v_2, \ldots, u_1v_{k-2}, u_0u_1, u_{d-1}u_d\}$ is an extreme edge-to-vertex geodetic set of G so that $g_{ev}(G) = k$. Therefore G is a perfect extreme edge-to-vertex geodesic graph. ■

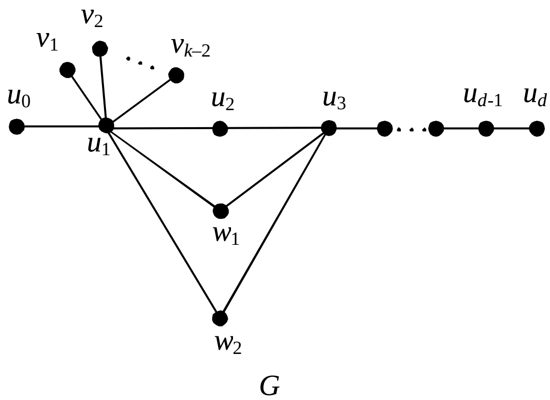

Figure 2.4(d)

For every connected graph, *rad* $G \leq$ *diam* $G \leq 2$ *rad* G. Ostrand[7] showed that every two positive integers a and b with $a \leq b \leq 2a$ are realizable as the radius and diameter, respectively, of some connected graph. Now, Ostrand's theorem can be extended to extreme to edge-to-vertex geodesic graphs.

Theorem 2.14. For positive integers r, d and $l \geq 3$ with $r<d\leq 2r$, there exists a perfect extreme edge-to-vertex geodesic graph G with *rad* $G = r$, *diam* $G = d$ and $g_{ev} = l = E(e)$.

Proof. When $r = 1$, let $G = K_{1,\,l}$. Then $d = 2$ and by Theorem 2.6, $g_{ev}(G) = l$ and G is a perfect extreme edge-to-vertex geodesic graph.. Now, let $r \geq 2$. Construct a graph G with the desired properties as follows. Let C_{2r} : $v_1, v_2, \ldots, v_{2r}, v_1$ be a cycle of order $2r$ and let P_{d-r+1} : $u_0, u_1, u_2, \ldots, u_{d-r}$ be a path of order $d - r + 1$. Let H be the graph obtained from C_{2r} and P_{d-r+1} by identifying v_1 in C_{2r} and u_0 in P_{d-r+1}. Now, add $(l - 3)$ new vertices $w_1, w_2, \ldots, w_{l-3}$ to H and join each vertex w_i $(1 \leq i \leq l - 3)$ to the vertex u_{d-r-1} and join the vertices v_r and v_{r+2} and obtain the graph G of Figure 2.5. Then *rad* $G = r$ and *diam* $G = d$. Let $S_e = \{v_rv_{r+1}, v_{r+1}v_{r+2}, u_{d-r-1}u_{d-r}, u_{d-r-1}w_1, u_{d-r-1}w_2, \ldots, u_{d-r-1}w_{l-3}\}$ be the set of l extreme edges of G. Let $S_1 = S_e - \{v_rv_{r+1}\}$ and $S_2 = S_e - \{v_{r+1}v_{r+2}\}$.

Then by Theorem 1.1**,** either S_1 or S_2 is a subset of every extreme edge-to-vertex geodetic set of G. It is clear that neither S_1 nor S_2 is an extreme edge-to-vertex geodetic set of G and so $g_{ev} \geq l$. However, S_e is an extreme edge-to-vertex geodetic set of G so that that $g_{ev} = l$. Therefore G is a perfect extreme edge-to-vertex geodesic graph. ■

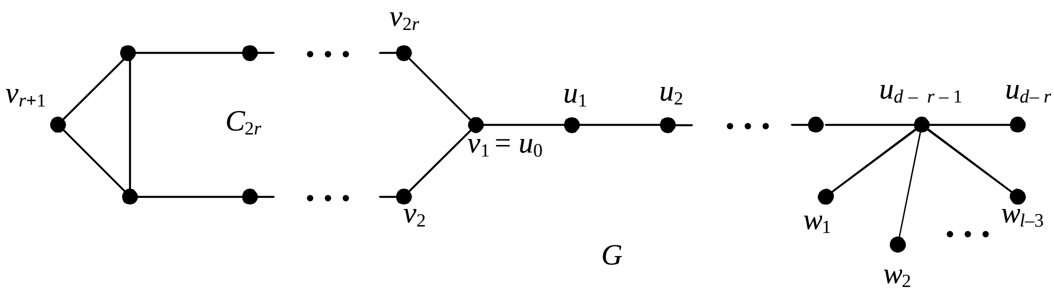

Figure 2.5

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